Multi-Contact Inertial Estimation and Localization in Legged Robots

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Introduction

RQ1 — How can robots be aware of changes in their dynamic model?

RQ2 — How do we formulate the problem, so we can solve it efficiently?

Optimal Estimation with parametrized dynamics

Objective function aims to find the optimal initial state, inertial parameters and disturbances that explain a set of **past** observations while satisfying the system's **dynamics**.

$$\min_{\mathbf{x}_s,\mathbf{w}_s,\boldsymbol{\theta}} \frac{1}{2}\|\mathbf{x}_0 \ominus \bar{\mathbf{x}}_0\|^2_{\boldsymbol{\Sigma}^{-1}_{\mathbf{x}_0}} + \frac{1}{2}\|\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}\|^2_{\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}}} + \frac{1}{2}\sum_{k=0}^{N-1}\|\mathbf{w}_k\|^2_{\boldsymbol{\Sigma}^{-1}_{\mathbf{w}_N}} + \frac{1}{2}\sum_{k=1}^{N}\|\hat{\mathbf{z}}_k \ominus \mathbf{h}(\mathbf{x}_k;\boldsymbol{\theta}|\hat{\mathbf{u}}_k)\|^2_{\boldsymbol{\Sigma}^{-1}_{\mathbf{z}_k}}$$
$$\text{s.t.}\quad \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k;\boldsymbol{\theta}|\hat{\mathbf{u}}_k) \oplus \mathbf{w}_k$$

Contributions

Inertial parameters parametrization	Analytical Derivatives	Multiple-shooting Solver
Smooth manifold with that guarantees full physical consistency Parametrization based on the SVD	**Algorithm for analytical derivatives of hybrid dynamics** Development of derivatives w.r.t. inertial parameters; for free, contact and impulse dynamics	**Multiple shooting algorithm for parametrized DDP with hybrid contact dynamics** Enables the resolution of intricate problems

DDP with parametrized dynamics

Optimal estimation problem is solved efficiently through a parametrized Riccati recursion.

$$\delta\mathcal{V}(\delta\mathbf{x};\delta\boldsymbol{\theta}|\hat{\mathbf{u}},\hat{\mathbf{z}}) \simeq \min_{\delta\mathbf{w}} \frac{1}{2}\begin{bmatrix}1\\ \delta\mathbf{x}\\ \delta\mathbf{w}\\ \delta\boldsymbol{\theta}\end{bmatrix}^{\intercal}\begin{bmatrix} & \mathbf{Q}_{\mathbf{x}}^{\intercal} & \mathbf{Q}_{\mathbf{w}}^{\intercal} & \mathbf{Q}_{\boldsymbol{\theta}}^{\intercal}\\ \mathbf{Q}_{\mathbf{x}} & \mathbf{Q}_{\mathbf{xx}} & \mathbf{Q}_{\mathbf{xw}} & \mathbf{Q}_{\mathbf{x}\boldsymbol{\theta}}\\ \mathbf{Q}_{\mathbf{w}} & \mathbf{Q}_{\mathbf{xw}}^{\intercal} & \mathbf{Q}_{\mathbf{ww}} & \mathbf{Q}_{\mathbf{w}\boldsymbol{\theta}}\\ \mathbf{Q}_{\boldsymbol{\theta}} & \mathbf{Q}_{\mathbf{x}\boldsymbol{\theta}}^{\intercal} & \mathbf{Q}_{\mathbf{w}\boldsymbol{\theta}}^{\intercal} & \mathbf{Q}_{\boldsymbol{\theta\theta}}\end{bmatrix}\begin{bmatrix}1\\ \delta\mathbf{x}\\ \delta\mathbf{w}\\ \delta\boldsymbol{\theta}\end{bmatrix}$$

We obtain a control policy that provides the optimal search direction for the disturbance, parameters and initial state.

$$\delta\mathbf{w} = -\mathbf{k} - \mathbf{K}\delta\mathbf{x} - \mathbf{P}\delta\boldsymbol{\theta}$$
$$\delta\boldsymbol{\theta} = -\mathbf{k}_{\boldsymbol{\theta}} - \mathbf{K}_{\boldsymbol{\theta}}\delta\mathbf{x}_0,\quad \delta\mathbf{x}_0 := -\mathcal{V}^{-1}_{\mathbf{x}\mathbf{x}_0}\mathcal{V}_{\mathbf{x}_0}$$

- The presence of singularities in the parametrised dynamics is handled by a Null-Space parametrization.

$$\mathbf{Y}^{\intercal}\mathcal{V}^{\bullet}_{\boldsymbol{\theta\theta}}\mathbf{Y}\delta\boldsymbol{\theta}_{\mathbf{y}} = -\overbrace{\mathbf{Y}^{\intercal}\mathcal{V}^{\bullet}_{\boldsymbol{\theta}}}^{\mathbf{k}_{\boldsymbol{\theta}\mathbf{y}}} - \overbrace{\mathbf{Y}^{\intercal}\mathcal{V}^{\bullet\,\intercal}_{\mathbf{x}\boldsymbol{\theta}}}^{\mathbf{K}_{\boldsymbol{\theta}\mathbf{y}}}\delta\mathbf{x}_0$$

- The proposed multiple-shooting algorithm distributes the non-linearities, increasing the basin of attraction. This allows us to solve intricate problems consisting of challenging maneuvers.

$$\mathbf{w}^{+}_{k} = \mathbf{w}_k + \alpha\delta\mathbf{w}_k,$$
$$\mathbf{x}^{+}_{k+1} = \mathbf{x}_{k+1} \oplus \alpha\delta\mathbf{x}_{k+1},$$
$$\bar{\mathbf{f}}^{+}_{k+1} = \mathbf{f}(\mathbf{x}^{+}_{k}, \mathbf{w}^{+}_{k}; \boldsymbol{\theta}^{+}_{k}) \ominus \mathbf{x}^{+}_{k+1}$$

Exponential Eigenvalue parametization

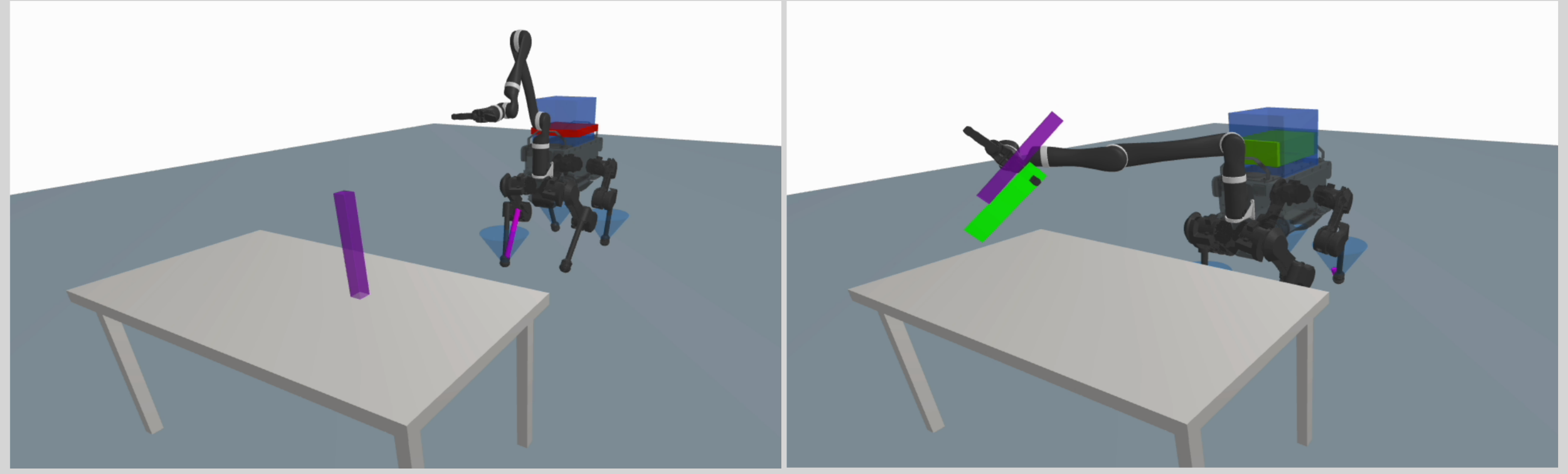
- Derivatives obtained from the joint torque regressor matrix

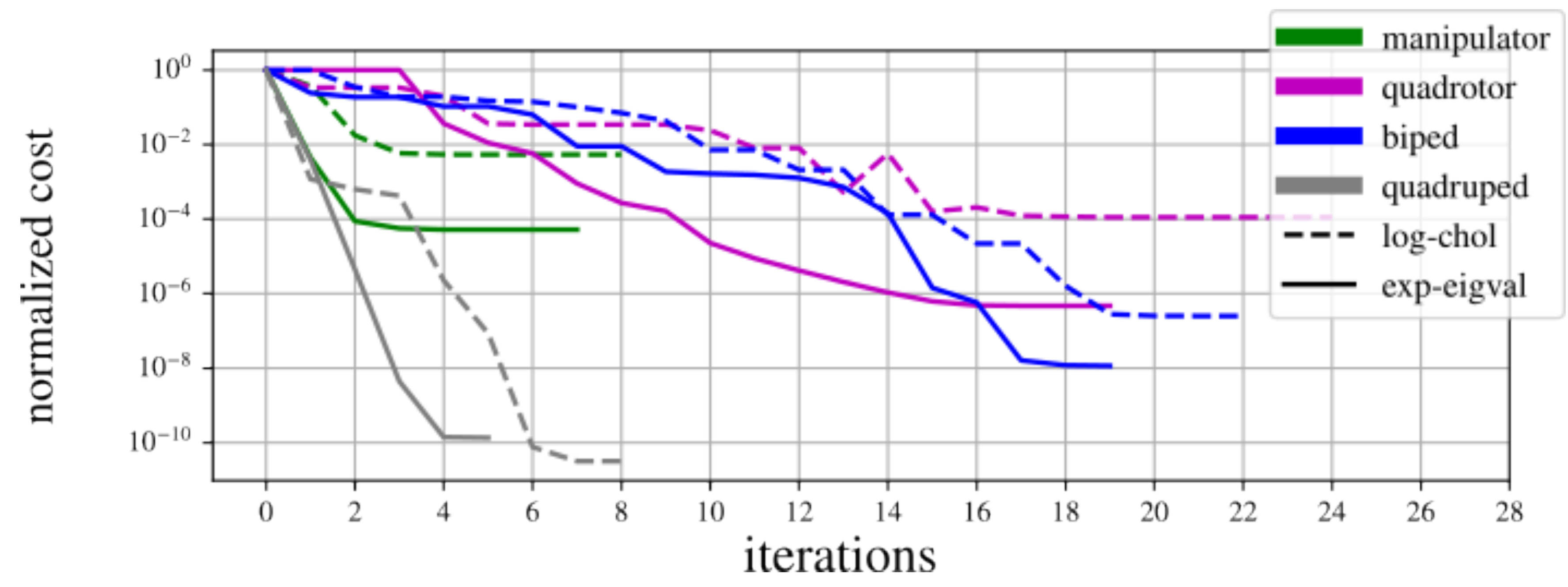
$$\frac{\partial \mathrm{FD}}{\partial\boldsymbol{\theta}} = \mathbf{M}(\mathbf{q})^{-1}\frac{\partial \mathrm{ID}}{\partial\boldsymbol{\pi}}\frac{\partial\boldsymbol{\pi}}{\partial\boldsymbol{\theta}} = \mathbf{M}(\mathbf{q})^{-1}\mathbf{Y}(\mathbf{q},\mathbf{v},\dot{\mathbf{v}})\frac{\partial\boldsymbol{\pi}}{\partial\boldsymbol{\theta}}$$

- Our parametrisation of inertial parameters shows lower non-linearities, higher convergence rates and provides with a direct physical meaning.

$$\mathbf{I}_c = \mathbf{R}\,\mathrm{diag}\,(\mathbf{PL})\,\mathbf{R}^{\intercal}\qquad m = \exp(\sigma_m),\qquad \mathbf{L} = \left[\exp(\sigma_x)\ \exp(\sigma_y)\ \exp(\sigma_z)\right]^{\intercal}$$

Framework capable of estimating unknown objects





Experimental validation on Go1

Our framework is validated experimentally showing the relevance of a correct estimation of inertial parameters and the importance of the dynamics in the localisation of dynamic motions.